MA 5629 – Numerical PDEs
Michigan Technological University
Fall 2016
Homework #1

1. Consider the 1D heat equation for $x \in [0, 1]$,

$$\begin{aligned} u_t &= u_{xx}, \\ u(0, x) &= 1 + \cos 2\pi x + 4 \cos 6\pi x \\ u(t, 0) &= 6, \quad u_x(t, 1) = 1 \end{aligned}$$

   (a) Write down an implicit finite difference scheme that is first-order in time and second order in space. Explain how the difference formula for the Neumann boundary condition is derived.

   (b) What is the local truncation error for this scheme?

   (c) Implement this scheme in a language of your choice, and demonstrate that you achieve the expected order of convergence (both space and time) for $t \in [0, 5]$. Use an error norm that you think is appropriate.

2. Consider Burgers' equation in 1D on the interval $[-2, 2]$,

$$\begin{aligned} u_t + uu_x &= 0, \\ u(0, x) &= \begin{cases} 1 & x < 0 \\ 0 & x > 0 \end{cases} \end{aligned}$$

   (a) Derive a finite-volume scheme that is vertex-centered, and uses piecewise constant (0^th-order) reconstruction. You can assume that $u_i^n \geq 0$ for all j and n. Be sure to explain how you handle the boundary conditions.

   (b) Implement this scheme in a language of your choice, and demonstrate that your code converges to the correct weak solution, and that you achieve the expected order of convergence (both space and time) for $t \in [0, 2]$. Use an error norm that you think is appropriate