MA 5629 – Numerical PDEs
Michigan Technological University
Fall 2016
Homework #3, Due 11/11

1. Motivated by question 0.x.4 in the text, define a mesh:

$$0 = x_0 < x_{\frac{1}{2}} < x_1 < x_{\frac{3}{2}} < x_2 < \cdots < x_{n-\frac{1}{2}} < x_n = 1,$$

and grid spacing, $h_j = x_j - x_{j-1}.j = 1, \ldots, n$. The basis functions associated with mesh nodes satisfy

$$\phi_j(x) = \begin{cases} \dfrac{2}{h_j^2}(x - x_{j-1})(x - x_{j-1/2}), & \text{if } x_{j-1} \le x < x_j \\ \dfrac{2}{h_{j+1}^2}(x_{j+1} - x)(x_{j+1/2} - x), & \text{if } x_j \le x < x_{j+1} \\ 0, & \text{otherwise} \end{cases}, \quad j = 1, 2, \ldots, n,$$

and those associated with the midpoints satisfy

$$\phi_{j-\frac{1}{2}}(x) = \begin{cases} \dfrac{4}{h_j^2}(x_j - x)(x - x_{j-1}), & \text{if } x_{j-1} \le x < x_j \\ 0, & \text{otherwise} \end{cases} \qquad j = 1, 2, \ldots, n.$$

Tasks:

- Compute a local stiffness matrix for these quadratic elements.
- Assemble a global stiffness matrix
- Solve our canonical problem,

$$-u''(x) = \sin 2\pi x, \quad u(0) = 0, \quad u'(1) = 0$$

- Conduct a numerical convergence study with an appropriate error norm. Comment on whether the observed convergence matches analysis?

2. (0.x.2.i) Give the weak formulation of the two-point boundary value problem

$$\begin{aligned} -u'' + u &= f, \qquad x \in [0, 1] \\ u(0) &= u'(1) = 0 \end{aligned}$$