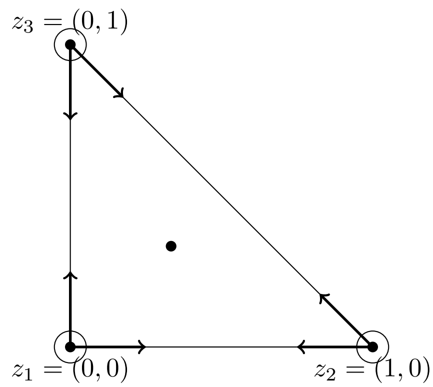
MA 5629 – Numerical PDEs
Michigan Technological University
Fall 2016
Homework #4, Due 12/5

1. Consider "local" cubic Hermite elements, i.e. the nodal parameters of each triangle angle are invariant with respect to the triangle, (K, P, N), where K is the standard reference triangle.



Let $N_7(f)$ denote the nodal parameter that measures the gradient of f at z_2 in the direction of z_1, and let $N_8(f)$ denote the nodal parameter that measures the gradient of f at z_2 in the direction of z_3. Let $\{\phi_1, \ldots, \phi_{10}\}$ be the basis dual to N,

Construct part of the local stiffness matrix involving ϕ_7 and ϕ_8 for the variational problem

$$a(u, v) = \nabla u \cdot \nabla v,$$

i.e., compute $a(\phi_i, \phi_j)$ for $i, j = \{7, 8\}$.

2. Consider the cubic hermite element, $(\hat{K}, \hat{P}, \hat{N})$, where $\hat{K}$ is the triangle with vertices $h(0, 1)$, $h(2, 2)$ and $h(1, 4)$, and where h denotes some mesh discretization. (i) Find an affine transformation that maps the reference element. (ii) Use your computation from above to compute $a(\hat{\phi}_i, \hat{\phi}_j)$ for $i, j = \{7, 8\}$.

3. Show that these local hermite cubic elements are C_0 elements.

4. Show that these local hermite cubic elements are not C_1 elements.